Chapter I: Basic Concepts

1.1 Introduction to the Computational Methods
1.2 Introduction to Second-order PDE
1.3 Classification of Second-order PDE
1.4 Cauchy Problem and Cauchy-Kowalewsky Theorem
1.5 Well-posed and Ill-posed problems
1.6 Qualitative Properties of Elliptic, Parabolic and Hyperbolic PDE

Chapter II: Finite-Difference Calculus

2.1 Taylor Series Expansion
2.2 Order Symbol and Gauge Function
2.3 Truncation Error
2.4 Finite Difference Approximation to a Derivative
2.5 Higher-order approximation
2.6 Difference Operator Theory
2.7 Implicit Finite Difference Formula

Chapter III: Difference Method for Parabolic PDE's

3.1 Explicit Finite-Difference Approximation to the Heat Equation
3.2 Concept of Convergence
3.3 Worked Example and Comparison with Analytical Solutions
3.4 Propagation Speed of Disturbance and its Relation to Stability
3.5 Implicit Finite-Difference Approximation to the Heat Equation
3.6 Thomas Algorithm
3.7 Crank-Nicalson Method
3.8 Keller-Box Method
3.9 Block-Tridiagonal System
3.10 Treatment of Derivative Boundary Condition
3.11 Fortran Hints in Programming
3.12 Roundoff Error and Worked Example
3.13 Diagonal Dominancy

Chapter IV: Stability of Finite Difference Method

4.1 Exact Solution of Heat Eq. by Fourier Series
4.2 Exact Solution of the Finite-Difference Approximation of the Heat Equation
4.3 Definition of Stability
4.4 Von-Neumann Stability Analysis
4.5 Gerschgorin Circle Theorem
4.6 Matrix Stability Method
4.7 Examples

Chapter V: Finite-Difference Methods for Elliptic Equations

5.1 Introduction
5.2 Finite-Difference Approximation to Poisson's Equation
5.3 Existance and Uniqueness of the Solution
5.4 Jocobi Iteration and its Rate of Convergence
5.5 Gauss-Seidal Iteration
5.6 SOR or Successive Over-Relaxation Method

Chapter VI: Finite Volume Method (FVM)

6.1 Divrgence Form of the Governing Equations
6.2 Convection Diffusion Equation
6.3 Discretization and Approximations
6.4 FVM for Steady One-Dimensional Convection-Diffusion Problems
6.5 Upwind Approximation
6.6 Hybrid Method
6.7 FVM for Steady Two-Dimensional Convection-Diffusion Problems
6.8 FVM for Navier-Stokes Equations
6.9 The Staggered Grid
6.10 Semi-Implicit Method for Pressure-Linked Equation (SIMPLE)

Chapter VII: Hyperbolic Equations (Theory)

7.1 Constant Coefficient Advection Equation (1-D Wave Eq.)
7.2 Initial-Value Problem
7.3 Initial-Boundary Value Problem
7.4 Characteristic Theory for Linear and Quasi-linear Wave Equations
7.5 Examples
7.6 Conservation Law Equation
7.7 Characteristic Intersection and Shock Formation
7.8 Weak or Generalized Solution
7.9 Rankine-Hugoniot Condition
7.10 Gas Dynamics Shock Wave

Chapter VIII: Numerical Solution of Hyperbolic Equations

8.1 Method of Characteristics
8.2 Examples
8.3 Explict Finite Difference for 1-D Wave Equation
8.4 FTFS, FTBS, FTCS, Lax-Fridrichs Method, CTCS, Lax-Wendroff Methods
8.5 Von-Neumann Stability Analysis (Revisited)
8.6 Stability Analysis of above Methods
8.7 CFL Stability and its Physical Significance
8.8 Implicit Finite Difference Methods
8.9 Scalar Conservation Laws Formulation
8.10 Monton, Upwind and Downwind Methods
8.11 Dispersion and Dissipation of PDE
8.12 Dispersion and Dissipation of FDE

TEXT: Class Notes

Grading: The following weights will be used for grading:

Homeworks	35%
Midterm	25%
Final Exam	40%
Total	100%