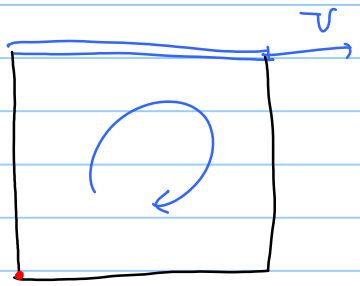
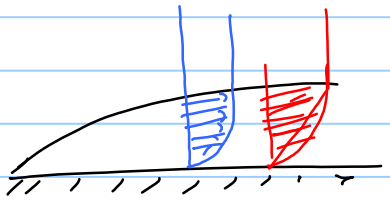
Note Title 5/16/2020

CFDI - شنبه ۹۹/۲/۲۷





$P(0,0) = 0$ ✓

تئوری معادلات هذلولوی :

به شکل سره :

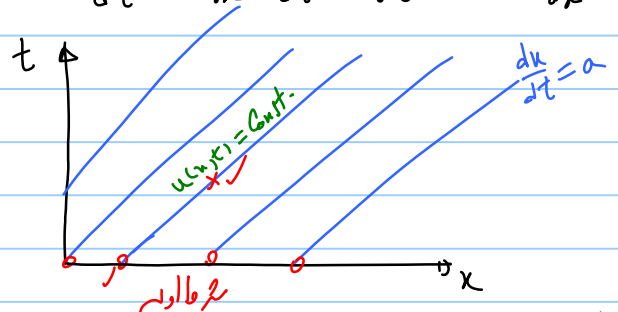
$$a u_x + u_t = 0 \quad , \quad a = Const. \qquad (1)$$

$u(x,t) = ?$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t}$$

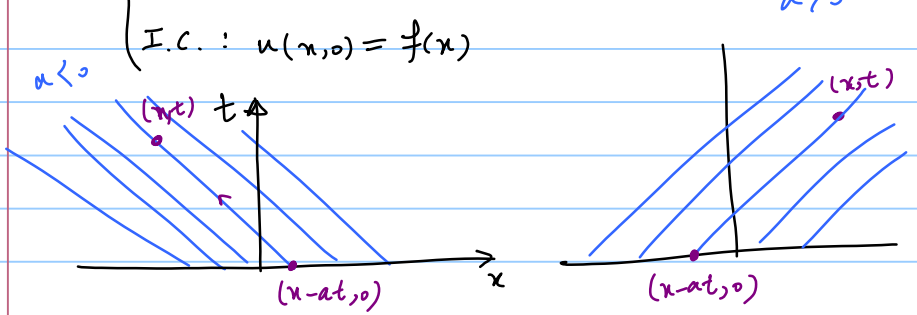
اگر $\frac{dx}{dt} = a$ باشد :

$$\Rightarrow \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t} = a\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$$



مسئله مقدار اولیه (مسئله کوشی) :

$$\checkmark \begin{cases} a u_x + u_t = 0 \\ I.C. : u(x,0) = f(x) \end{cases} \qquad -\infty < x < \infty$$



$\frac{dx}{dt} = a \Rightarrow x = at + c \Rightarrow$ خطوط مشخصه

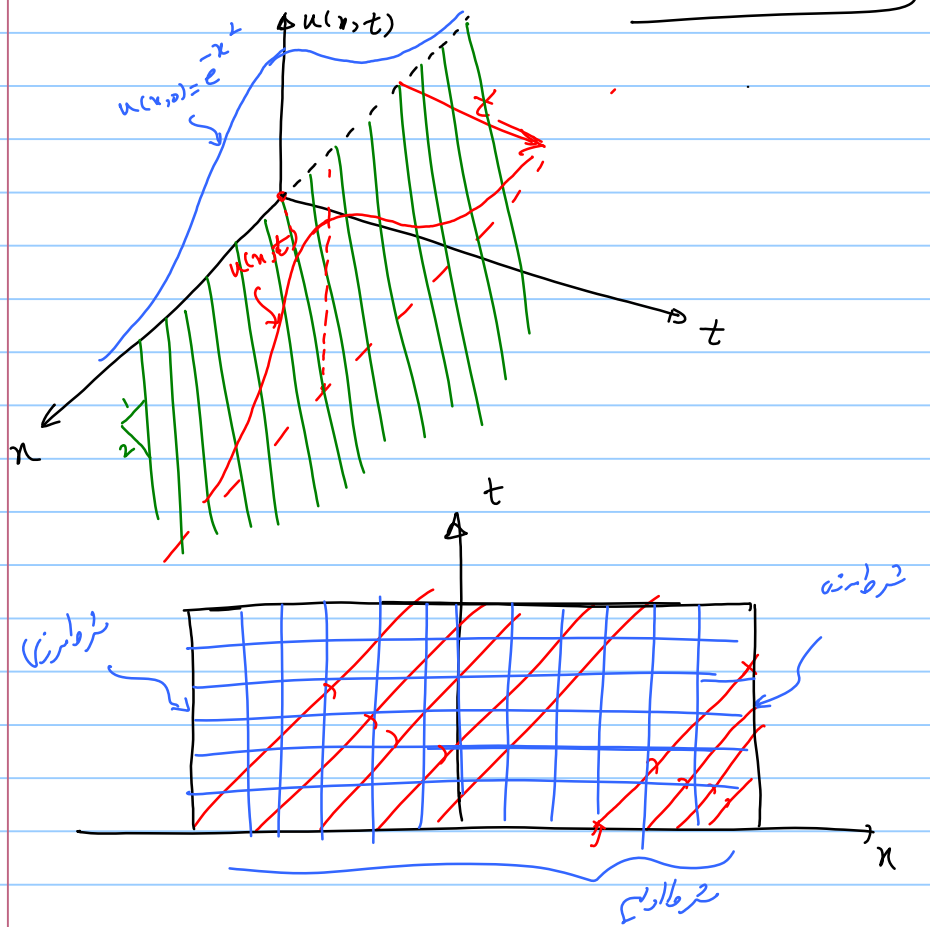
$\Rightarrow u(x,t) = u(x-at, 0) = f(x-at)$

مثال:

$$\begin{cases} 2u_x + u_t = 0 & -\infty < x < \infty \\ u(x,0) = e^{-x^2} \end{cases}$$

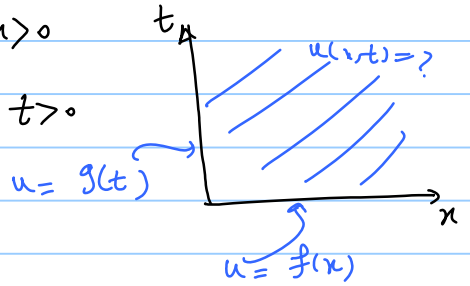
معادله مشخصه: $\frac{dx}{dt} = a = 2 \Rightarrow x = 2t + x_0$

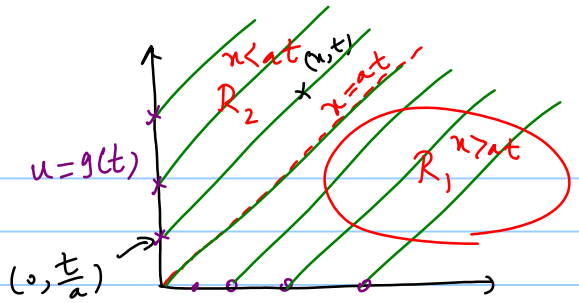
$u(x,t) = u(x-at, 0) = u(x-2t, 0) = e^{-(x-2t)^2}$



مسئله مقدار اولیه و مرزی:

$$\begin{cases} au_x + u_t = 0 & x>0,\ t>0 \\ u(x,0) = f(x) & x>0 \\ u(0,t) = g(t) & t>0 \end{cases}$$





برای حالتی $a>0$:

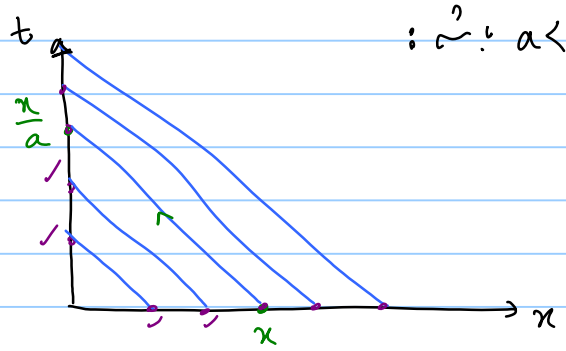
برای ناحیه R_1 :

جواب کلی: $u(x,t) = u(x-at, 0) = f(x-at)$

برای ناحیه R_2 :

$$u(x,t) = u\left(0, t-\frac{x}{a}\right) = g\left(t-\frac{x}{a}\right)$$

$$\Rightarrow u(x,t) = \begin{cases} f(x-at) & x>at \\ g\left(t-\frac{x}{a}\right) & x<at \end{cases}$$



حالتی که $a<0$ باشد:

$$IC: \quad u(x,0) = f(x)$$

$$BC: \quad u(0,t) = g(t)$$

$$\Rightarrow u\left(0, \frac{x_0}{|a|}\right) = g\left(\frac{x_0}{|a|}\right)$$

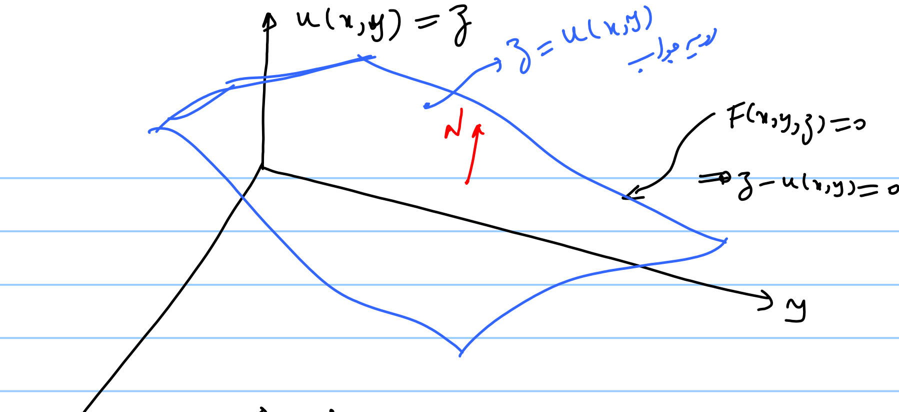
$\Rightarrow$

شرط سازگاری $f(x_0) = g\left(\frac{x_0}{|a|}\right)$

معادلات شبه خطی:

$$a(x,y,u)\,u_x + b(x,y,u)\,u_y = c(x,y,u)$$

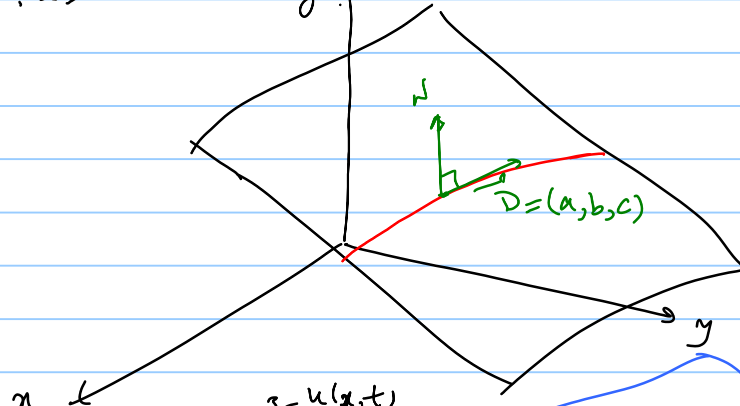
معادله‌ی فوق با شرط اولیه مانند $u(x,0) = f(x)$ یک مسئله کوشی است.



$$\vec{N} = \vec{\nabla}(u - z) = (u_x, u_y, -1)$$

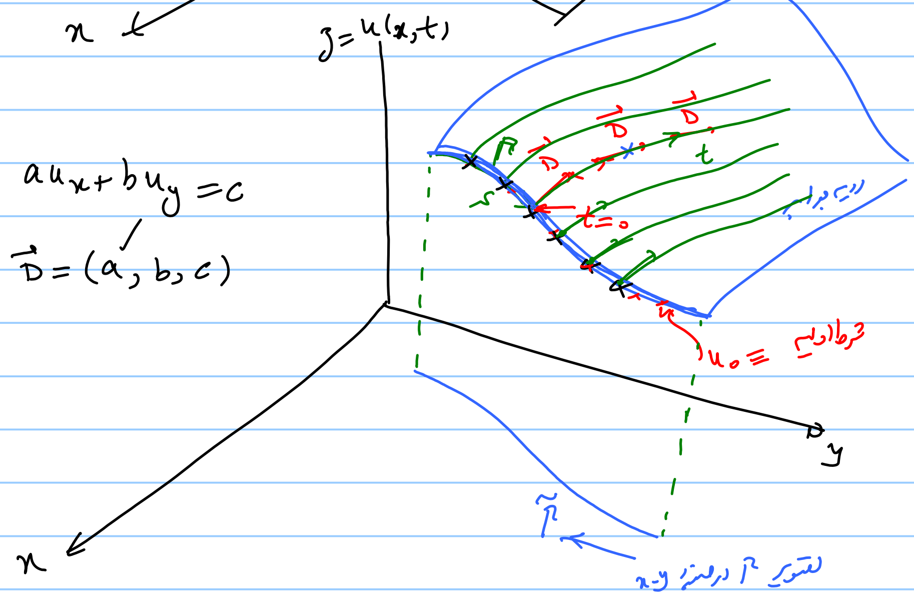
$$\vec{N} \cdot \underbrace{(a, b, c)} = a u_x + b u_y - c = 0$$

بردار مماس بر رویه جواب



$$a u_x + b u_y = c$$

$$\vec{D} = (a, b, c)$$



$$\begin{cases} \dfrac{dx}{dt} = \dot{x} = a(x, y, z) & x(0) = f(s) \\ \dfrac{dy}{dt} = \dot{y} = b(x, y, z) & y(0) = g(s) \\ \dfrac{dz}{dt} = \dot{z} = c(x, y, z) & z(0) = h(s) \end{cases}$$

با حل دو معادله اول $x = x(s,t)$, $y = y(s,t)$ [illegible] می‌شوند

$\Rightarrow s = s(x,y)$, $t = t(x,y)$

از سه معادله: $z = z(s,t)$ مشخص می‌شود.

$$\Rightarrow z = z(x,y) \checkmark = u(x,y)$$

با تحت چه شرایطی از $x = x(s,t)$ و $y = y(s,t)$ می‌توان

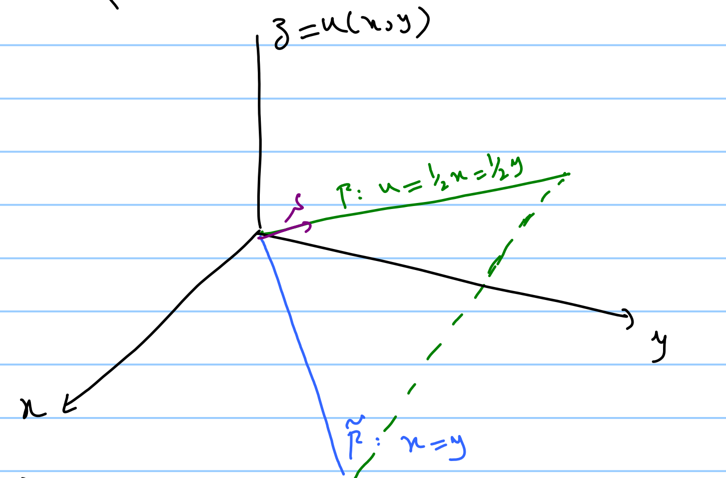
$s = s(x,y)$ و $t = t(x,y)$ را به دست آورد؟

هنگامی که ژاکوبین تبدیل مخالف صفر باشد.

$$J = \frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix} \neq 0$$

مثال:

$$\begin{cases} u\, u_x + u_y = 1 \\ IC: \quad u = \frac{1}{2} x \qquad \text{on } \tilde{\Gamma}: \ x = y \end{cases}$$



به ازای $x_0 = s \Rightarrow y_0 = s \Rightarrow u_0 = \frac{1}{2} s$

$$\begin{cases} \dot{x} = a(x,y,z) = u & x(0) = s \\ \dot{y} = b(x,y,z) = 1 & y(0) = s \\ \dot{z} = c(x,y,z) = 1 & z(0) = \frac{1}{2} s \end{cases}$$

$\dot{y} = 1 \ , \ y(0) = s \Rightarrow y = t + s$ (1)

$\dot{z} = 1 \ , \ z(0) = \frac{1}{2} s \Rightarrow z = t + \frac{1}{2} s$ (2)

$\dot{x} = u = z \Rightarrow \dot{x} = t + \frac{1}{2} s \ , \ x(0) = s$

$$\Rightarrow x = \frac{1}{2}t^2 + \frac{1}{2}st + s \qquad (3)$$

با توجه به روابط (1) و (3)

$$\Rightarrow t = 2\frac{y-x}{2-y} \quad , \quad s = \frac{2x-y^2}{2-y}$$

با جایگذاری در (2) جواب به دست می‌آید:

$$z = u(x,y) = \underbrace{2\frac{y-x}{2-y}}_{t} + \underbrace{\frac{1}{2}\left(\frac{2x-y^2}{2-y}\right)}_{\frac{1}{2}s}$$

$$\Rightarrow u(x,y) = \frac{2(y-x) + x - \frac{y^2}{2}}{2-y}$$

اگر یک مشتق ژاکوبین تبدیل به دست آید:

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2}r \\ 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1 - \frac{1}{2}s = 0 \Rightarrow s = 2$$

یعنی $s=2$ معادل $y=2$ است.