CFD I - اولین جلسه مجازی

یادآوری $\Rightarrow$

$$D = \frac{1}{h}\ln(1+\Delta)$$

$$D = -\frac{1}{h}\ln(1-\nabla)$$

$$D = \frac{2}{h}\sinh^{-1}\left(\frac{\delta}{2}\right)$$

$$D = \frac{2}{h}\cosh^{-1}(\mu)$$

یک
چگونگی استفاده از اپراتورها:

$$D = \frac{1}{h}\ln(1+\Delta) = \frac{1}{h}\left(\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \cdots\right) \leftarrow$$

$$\Delta = e^{hD} - 1 = hD + \frac{h^2}{2!}D^2 + \cdots \Rightarrow \Delta = O(h)$$

$$Df_n = \frac{1}{h}\left(\Delta - \frac{1}{2}\Delta^2\right)f_n + O(h^2)$$

$$\Rightarrow \underline{f'_n} = \frac{1}{h}\left(\Delta f_n - \frac{1}{2}\Delta^2 f_n\right)$$

$$= \frac{1}{h}\left[f_{n+1} - f_n - \frac{1}{2}(E-1)^2 f_n\right]$$

$$= \frac{1}{2h}\left(-f_{n+2} + 4f_n - 3f_n\right) + O(h^2)$$

$$D = \frac{2}{h}\sinh^{-1}\left(\frac{\delta}{2}\right)$$

$$\sinh^{-1}x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1\times 3}{2\times 4}\frac{x^5}{5} - \frac{1\times 3\times 5}{2\times 4\times 6}\frac{x^7}{7} + \cdots$$

$$\Rightarrow D = \frac{2}{h}\left\{\frac{\delta}{2} - \frac{\delta^3}{48} + \cdots\right\} = \frac{\delta}{h} - \frac{\delta^3}{24h} + \cdots$$

$$\Rightarrow Df_n = \frac{\delta}{h} f_n = \frac{E^{1/2} - E^{-1/2}}{h} f_n$$

$$\Rightarrow f_n' = \frac{f_{n+1/2} - f_{n-1/2}}{h} + O(h^2)$$

$$\delta = 2\sinh\left(\frac{hD}{2}\right) , \quad \mu = \cosh\left(\frac{hD}{2}\right)$$

$$\mu\delta = 2\sinh\left(\frac{hD}{2}\right)\cosh\left(\frac{hD}{2}\right) = \sinh(hD)$$

$$\Rightarrow D = \frac{1}{h}\sinh^{-1}(\mu\delta)$$

$$= \frac{1}{h}\left\{\mu\delta - \frac{1}{6}(\mu\delta)^3 + \cdots\right\} \quad (*)$$

$$\Rightarrow Df_n = \frac{1}{h}(\mu\delta)f_n = \frac{\mu}{h}(\delta f_n)$$

$$= \frac{\mu}{h}\left(f_{n+1/2} - f_{n-1/2}\right)$$

$$= \frac{1}{2h}\left[(f_{n+1} + f_n) - (f_n + f_{n-1})\right]$$

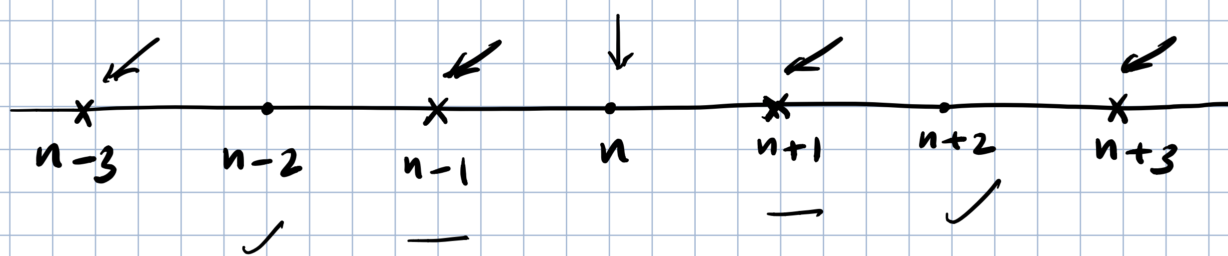
$$= \frac{f_{n+1} - f_{n-1}}{2h}$$

تعریف : $\bar{\delta} = \mu\delta \equiv$ Central Diff. operator

$$\bar{\delta} = \mu\left(E^{1/2} - E^{-1/2}\right) = \frac{1}{2}\left(E^{1} + E^{0} - E^{0} - E^{-1}\right)$$

$$= \frac{1}{2}\left(E - E^{-1}\right)$$

$$(*) \Rightarrow \quad D = \frac{1}{h}\left(\bar{\delta} - \frac{1}{6}\bar{\delta}^{3} + \cdots\right)$$



$$\mu^{2} = 1 + \frac{\delta^{2}}{4} \Rightarrow \mu^{-2}\mu^{2} = \mu^{-2}\left(1 + \frac{\delta^{2}}{4}\right)$$

$$\Rightarrow 1 = \mu\left(1 + \frac{\delta^{2}}{4}\right)^{-1/2}$$

$$= \mu\left(1 - \frac{\delta^{2}}{8} + \frac{3\delta^{4}}{128} - \frac{5\delta^{6}}{1024} + \cdots\right) \quad (**)$$

$$D = \frac{2}{h}\left\{\frac{\delta}{2} - \frac{\delta^{3}}{48} + \frac{3\delta^{5}}{1280} - \cdots\right)$$

$$hD = \delta - \frac{\delta^{3}}{24} + \frac{3\delta^{5}}{640} - \cdots \leftarrow$$

$$\Rightarrow hD = \mu \left\{ \delta - \frac{1}{3!}\delta^3 + \frac{1^2 \times 2^2}{5!}\delta^5 - \cdots \right\}$$

$$= \bar{\delta} \left\{ 1 - \frac{\delta^2}{3!} + \frac{2^2}{5!}\delta^4 - \frac{2^2 \times 3^2}{7!}\delta^6 + \cdots \right\}$$

$$D f_n = \frac{-f_{n+2} + 8 f_{n+1} - 8 f_{n-1} + f_{n-2}}{12 h} + O(h^4)$$

Implicit Finite Difference Formula: روابط تفاضل محدود ضمنی

$$hD = \mu\delta \left(1 - \frac{\delta^2}{6}\right) + O(h^5)$$

$$\left(1 + \frac{\delta^2}{6}\right)^{-1} = 1 - \frac{\delta^2}{6} + \cdots$$

$$\Rightarrow hD = \frac{\mu\delta}{1 + \frac{\delta^2}{6}} \longrightarrow$$ rational function or PADE Difference

$$\Rightarrow D f_n = \frac{1}{h}\left(\frac{\mu\delta}{1 + \delta^2/6}\right) f_n$$

$$\Rightarrow h\left(1 + \frac{\delta^2}{6}\right) D f_n = (\mu\delta) f_n + O(h^5)$$

$$\Rightarrow h\left[1 + \frac{1}{6}\left(E^{1/2} - E^{-1/2}\right)^2\right] D f_n = \frac{f_{n+1} - f_{n-1}}{2} + O(h^5)$$

$$\Rightarrow h\left[1+\frac{1}{6}\left(E-E^{-1}-2\right)\right]\underbrace{D f_n}_{f'_n} = \frac{f_{n+1}-f_{n-1}}{2} + O(h^5)$$

$$\Rightarrow \underbrace{f'_{n+1}+4f'_n+f'_{n-1} = \frac{f_{n+1}-f_{n-1}}{3h}}_{\text{روش فشرده یا روش هرمیتی}} + \underline{O(h^4)}$$

محاسن روش فشرده:

۱- با تعداد نقاط درگیر کمتر، دقت به مراتب بالاتر را می‌توان بدست آورد.

۲- چون جواب وابسته به کل ناحیه است (معادلات ضمنی حل می‌کنند)، حل فراگیر است (Global property)

فرم ماتریسی:

$$\begin{bmatrix} \ddots & \ddots & \ddots & & & 0 \\ & \ddots & \ddots & \ddots & & \\ & & 1 & 4 & 1 & \\ & & & \ddots & \ddots & \ddots \\ 0 & & & & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ f'_{i-1} \\ f'_i \\ f'_{i+1} \\ \vdots \end{bmatrix} = \frac{1}{3h}\begin{bmatrix} \vdots \\ f_i - f_{i-2} \\ f_{i+1}-f_{i-1} \\ f_{i+2}-f_i \\ \vdots \end{bmatrix}$$

$$h^2D^2 = \delta^2\left(1-\frac{\delta^2}{12}\right) + O(h^6)$$

$$\Rightarrow h^2D^2 = \frac{\delta^2}{1+\frac{\delta^2}{12}} + O(h^6)$$

$$\Rightarrow h^2\left(1+\frac{\delta^2}{12}\right)\underbrace{D^2 f_n}_{f_n''} = \delta^2 f_n$$

$$\Rightarrow f''_{n+1} + 10 f''_n + f''_{n-1} = \frac{12}{h^2}\left\{f_{n+1} - 2f_n + f_{n-1}\right\} + \underline{O(h^4)}$$

برخی نکات مهم در برنامه نویسی :

✓ الگوریتم توماس اغلب مورد نیاز است لذا بطور کامل مطالعه شود.

$$\cancel{\frac{A}{2}} \Rightarrow \underline{0.5 * A}$$

$$A^2 \longrightarrow \underline{\cancel{A**2}} \longrightarrow \underline{A*A}$$