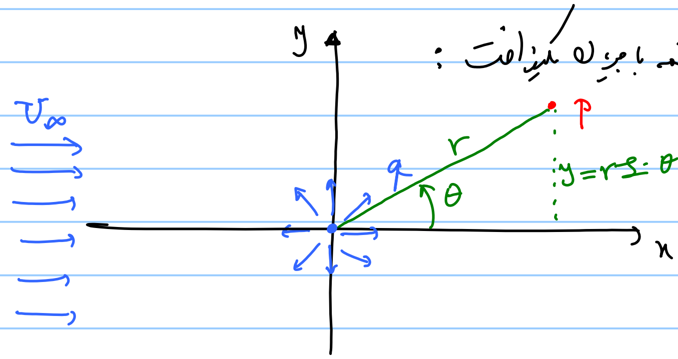
سیالات ۱ - یکشنبه ۱۱، ۳، ۹۹

ترکیب جریان چشمه با جریان یکنواخت :



$$\psi = \psi_{uniform} + \psi_{source}$$

$$= V_\infty y + \frac{q}{2\pi}\theta$$

$$= V_\infty r \sin\theta + \frac{q}{2\pi}\theta \quad ✓$$

$$\phi = V_\infty r \cos\theta + \frac{q}{2\pi}\ln r \quad ✓$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty \cos\theta + \frac{q}{2\pi r}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty \sin\theta$$

تعیین نقاط سکون

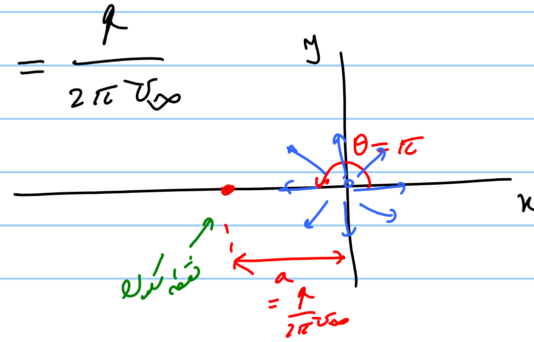
$$v_r = v_\theta = 0$$

$$v_\theta = 0 \Rightarrow -V_\infty \sin\theta = 0 \Rightarrow \theta = 0, \pi$$

$$\text{if } \theta = 0 \Rightarrow v_r = V_\infty + \frac{q}{2\pi r} \neq 0$$

$$\text{if } \theta = \pi \Rightarrow v_r = -V_\infty + \frac{q}{2\pi r} = 0$$

$$\Rightarrow r_s = a = \frac{q}{2\pi V_\infty}$$



لذا مقدار تابع جریان در نقطه رکود خواهد شد،

نقطه رکود: $r_s = \frac{q}{2\pi v_\infty}$, $\theta = \pi$

$$\psi_0 = \psi(r=a, \theta=\pi) = \frac{q}{2\cancel{\pi}}\cancel{\pi} = \frac{q}{2}$$

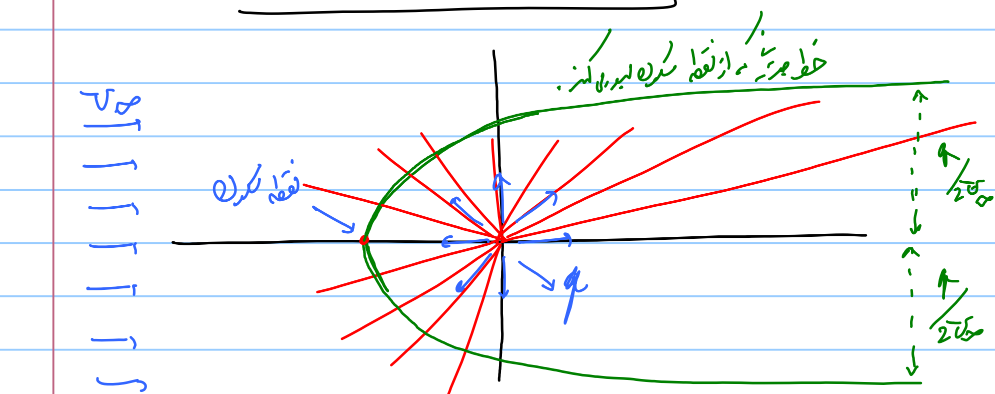
تابع جریان نقطه رکود

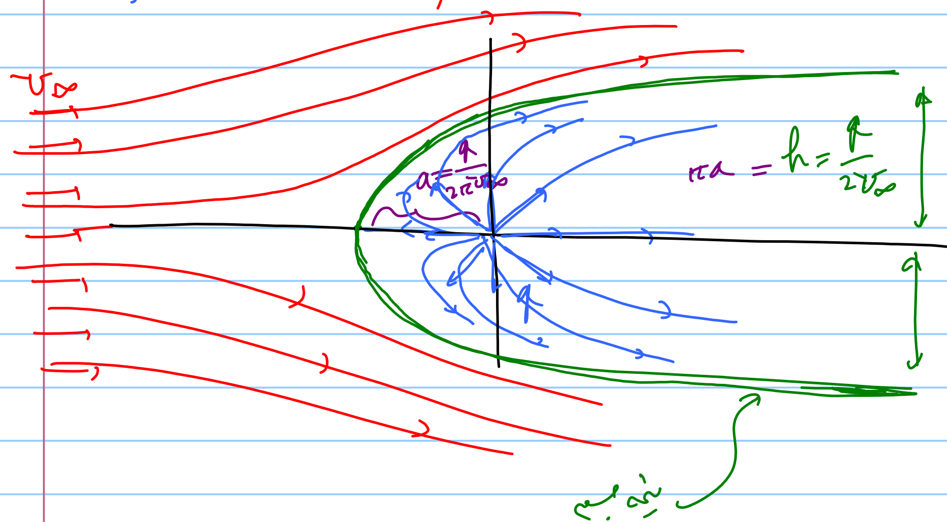
لذا خط جریانی که از نقطه رکود عبور می‌کند دارای رابطه زیر است:

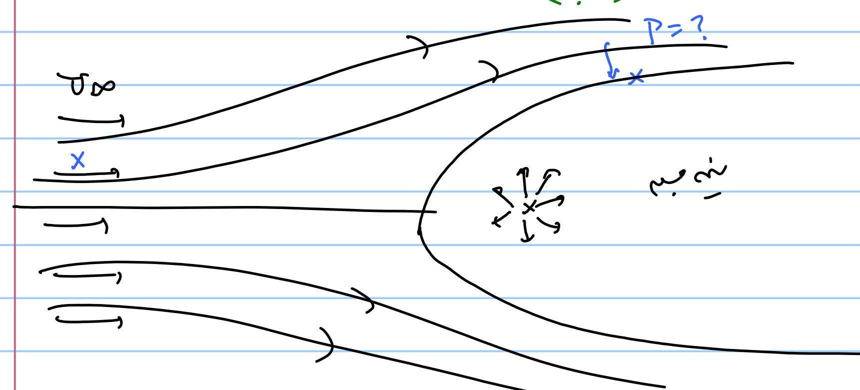
$$\psi = \psi_0 = \frac{q}{2} = \underbrace{v_\infty \overbrace{r\sin\theta}^{y} + \frac{q}{2\pi}\theta}$$

$$\Rightarrow \frac{q}{2} = v_\infty y_s + \frac{q}{2\pi}\theta$$

$$\Rightarrow \boxed{y_s = \frac{q}{2v_\infty}\left(1 - \frac{\theta}{\pi}\right)}$$







$$P_\infty + \tfrac{1}{2}\rho v_\infty^2 = P + \tfrac{1}{2}\rho v^2$$

$$v^2 = v_r^2 + v_\theta^2$$

$$\Rightarrow P - P_\infty = \tfrac{1}{2}\rho v_\infty^2 - \tfrac{1}{2}\rho v^2$$
$$= \underbrace{\tfrac{1}{2}\rho v_\infty^2}\left(1 - \frac{v^2}{v_\infty^2}\right)$$

$$C_p = \frac{P - P_\infty}{\tfrac{1}{2}\rho v_\infty^2} = 1 - \frac{v^2}{v_\infty^2}$$

ضریب فشار

$$v^2 = v_r^2 + v_\theta^2 = \left(v_\infty \cos\theta + \frac{q}{2\pi r}\right)^2 + \left(-v_\infty \sin\theta\right)^2$$
$$= v_\infty^2 + \frac{v_\infty q \cos\theta}{\pi r} + \left(\frac{q}{2\pi r}\right)^2$$

لذا: $h = \pi a = \dfrac{q}{2v_\infty} \Rightarrow \dfrac{q}{2\pi r} = v_\infty \dfrac{h}{\pi r}$

$$, \quad \frac{q}{\pi r} = v_\infty \frac{2h}{\pi r}$$

$$\Rightarrow \frac{v^2}{v_\infty^2} = 1 + \frac{2h}{\pi r}\cos\theta + \left(\frac{h}{\pi r}\right)^2$$

$$\Rightarrow C_p = -\frac{2h}{\pi r}\cos\theta - \left(\frac{h}{\pi r}\right)^2$$

ضریب فشار در هر نقطه از سیال دوبعدی

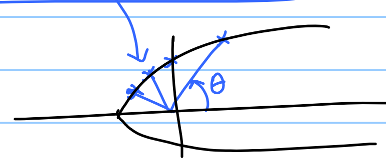
معادله شبه جسم مدور ۲

$$y_s = \frac{q}{2v_\infty}\left(1 - \frac{\theta}{\pi}\right) = \pi a\left(1 - \frac{\theta}{\pi}\right) = h\left(1 - \frac{\theta}{\pi}\right)$$

$r_s \sin\theta$

$$\Rightarrow \frac{h}{r_s} = \frac{\sin\theta}{\left(1 - \frac{\theta}{\pi}\right)} \Rightarrow \frac{h}{\pi r_s} = \frac{\sin\theta}{\pi - \theta}$$

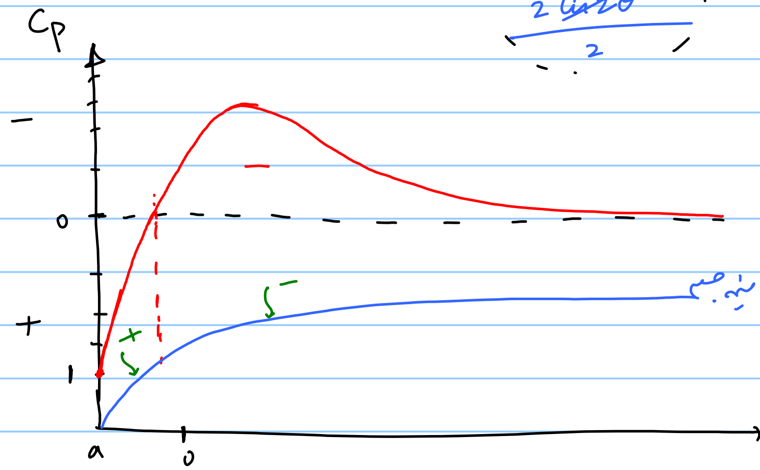
$$\Rightarrow C_{p_s} = -2\frac{\sin\theta}{\pi - \theta}\cos\theta - \left(\frac{\sin\theta}{\pi - \theta}\right)^2$$
$$= -\frac{\sin 2\theta}{\pi - \theta} - \frac{\sin^2\theta}{(\pi - \theta)^2}$$



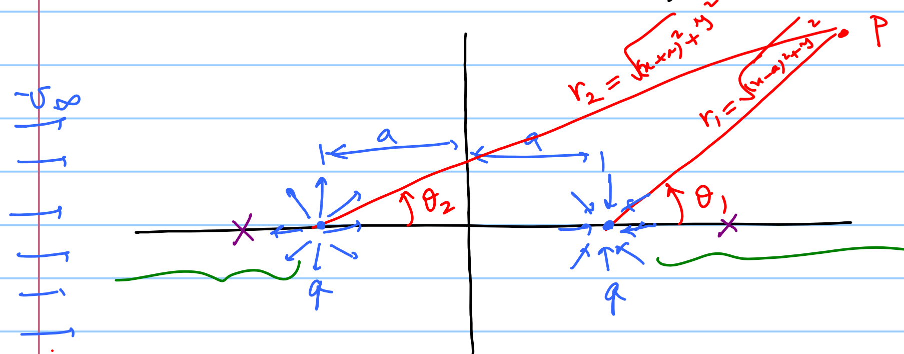
در نقطه سکون $\theta = \pi$

$$\Rightarrow C_p\Big|_{\theta=\pi} = 1 - \underbrace{\frac{2\cos 2\theta}{1}}_{2} - \frac{2\sin\theta\cos\theta}{2(\pi-\theta)}$$

$$= 2 - \left(\frac{\sin 2\theta}{2(\pi-\theta)}\right) = 1$$



تحلیل جریان حول جسم رانکین : (ترکیب جریان چشمه + چاه + جریان یکنواخت)



$$\phi = \phi_{\text{یکنواخت}} + \phi_{\text{چشمه}} + \phi_{\text{چاه}}$$

$$= v_\infty x + \frac{q}{2\pi}\ln\sqrt{(x+a)^2+y^2} - \frac{q}{2\pi}\ln\sqrt{(x-a)^2+y^2}$$

$$\psi = v_\infty y + \underbrace{\frac{q}{2\pi}\theta_2 - \frac{q}{2\pi}\theta_1}$$

$$\psi_{\text{چاه+چشمه}} = -\frac{q}{2\pi}\tan^{-1}\left(\frac{2a\overbrace{r\sin\theta}^{y}}{r^2-a^2}\right)$$

$$\Rightarrow \boxed{\psi = v_\infty y - \frac{q}{2\pi}\tan^{-1}\frac{2ay}{x^2+y^2-a^2}}$$

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} = v_\infty + \frac{q}{2\pi}\left[\frac{x+a}{(x+a)^2+y^2} - \frac{x-a}{(x-a)^2+y^2}\right]$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = \frac{q}{2\pi}\left[\frac{y}{(x+a)^2+y^2} - \frac{y}{(x-a)^2+y^2}\right]$$

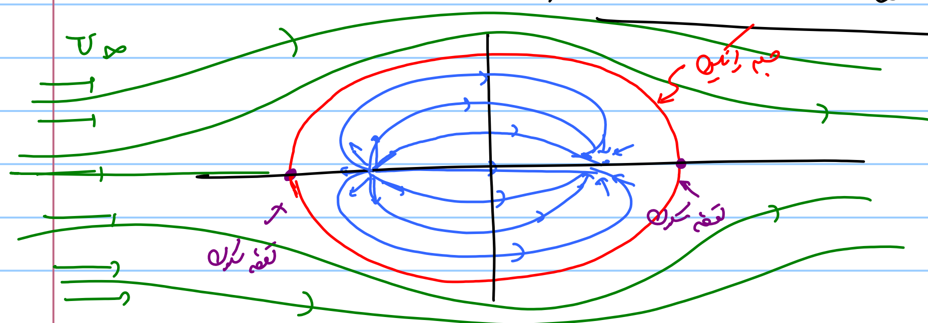
تعیین نقاط سکون :

$u = v = 0$

at $y=0 \Rightarrow v=0$ ✓

at $y=0$ if $u=0 \Rightarrow 0 = V_\infty + \frac{q}{2\pi}\left[\frac{1}{x+a} - \frac{1}{x-a}\right]$

$$\Rightarrow V_\infty = \frac{qa}{\pi(x^2-a^2)} \Rightarrow \boxed{x = \pm\sqrt{a^2 + \frac{qa}{\pi V_\infty}}}$$



$$\psi_0 = 0 \Rightarrow \boxed{0 = V_\infty y - \frac{q}{2\pi}\tan^{-1}\frac{2ay}{x^2+y^2-a^2}}$$

معادله خط جریانی که از نقاط سکون می‌گذرد.

$$\Rightarrow \boxed{\tan\frac{2\pi V_\infty y}{q} = \frac{2ay}{x^2+y^2-a^2}}$$

رابطه جسم رانکین

جریان حول استوانه :