Note Title 5/26/2020

سیالات ۱ - سه‌شنبه ۹۹٫۳٫۶

جریان پتانسیل : Potential Flow

اگر جریان غیرچرخشی باشد

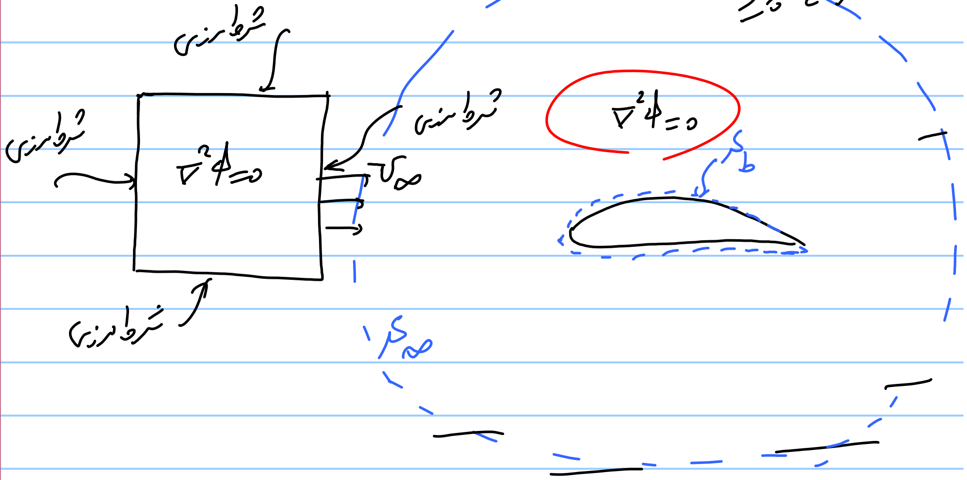
$\vec{\xi} = \vec{\nabla}\times\vec{V} = 0 \Rightarrow \vec{V} = \vec{\nabla}\phi$ ، $\phi \equiv$ تابع پتانسیل

$$\Rightarrow u=\frac{\partial\phi}{\partial x}, \quad v=\frac{\partial\phi}{\partial y}, \quad w=\frac{\partial\phi}{\partial z}$$

اگر جریان تراکم ناپذیر باشد،

پیوستگی : $\vec{\nabla}\cdot\vec{V}=0 \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\phi)=0$

$\Rightarrow \nabla^2\phi=0$ معادله حاکم بر تابع پتانسیل جریان تراکم ناپذیر

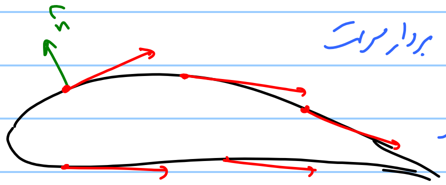


روی مرز ∞ ؛ S_∞ : $u=V_\infty$ ، $v=0$

$\Rightarrow \phi_\infty = V_\infty x$ ← شرط مرزی در ∞

$\Rightarrow v_n=0$

$\Rightarrow \frac{\partial\phi}{\partial n}=0$



شرط مماس بودن بردار سرعت بر جسم صلب

شرط مرزی روی جسم (شرط مماس بودن سرعت)

با درآوردن تابع جریان : ψ

$$u = \frac{\partial \psi}{\partial y} \,, \quad v = -\frac{\partial \psi}{\partial x}$$

$$\vec{\xi} = \vec{\nabla} \times \vec{V} = 0 \xrightarrow{\text{دو بعدی}} \xi_z = 0 = \underline{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \Rightarrow \underline{\nabla^2 \psi = 0}$$

در دستگاه مختصات استوانه‌ای :

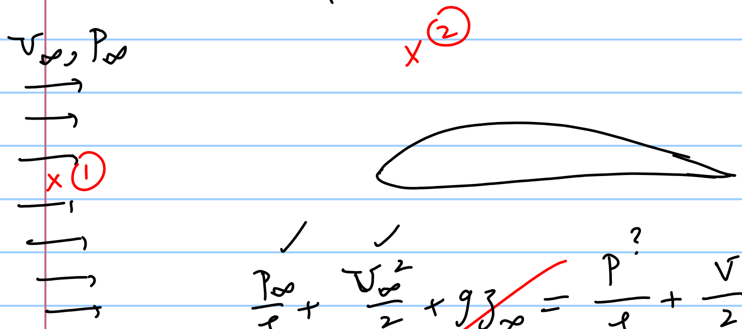
$$\nabla^2 \phi = 0 \Rightarrow \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

$$\nabla^2 \psi = 0 \Rightarrow \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0$$

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

نحوه تعیین میدان فشار، پس از بدست آوردن مولفه‌های سرعت :



$$\overset{\checkmark}{\frac{P_\infty}{\rho}} + \overset{\checkmark}{\frac{V_\infty^2}{2}} + \cancel{g z_\infty} = \frac{\overset{?}{P}}{\rho} + \frac{V^2}{2} + \cancel{g z}$$

$$\Rightarrow \underline{P = P_\infty + \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2}$$

$$V^2 = u^2 + v^2 = v_r^2 + v_\theta^2$$

با توجه به اینکه معادله لاپلاس برای جریان پتانسیل خطی است ($\nabla^2\phi=0$) لذا اگر ϕ_1 ، ϕ_2 ، ..... ، ϕ_n جواب $\nabla^2\phi=0$ باشند هر ترکیب خطی از آنها نیز جواب است

$$c_1\phi_1+c_2\phi_2+\dots+c_n\phi_n \quad \checkmark$$

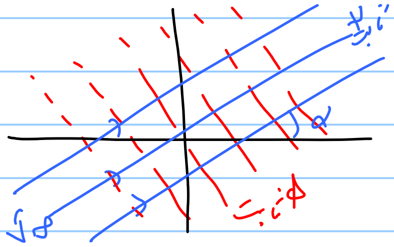
جریانهای اساسی : Basic Flows

(جریان یکنواخت با زاویه α و V∞) $\phi=(V_\infty \cos\alpha)\,x+(V_\infty \sin\alpha)\,y$ (1

$$\nabla^2\phi=0 \Rightarrow \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \Rightarrow 0+0=0 \quad \checkmark$$

$$u=\frac{\partial\phi}{\partial x}=V_\infty\cos\alpha$$

$$v=\frac{\partial\phi}{\partial y}=V_\infty\sin\alpha$$

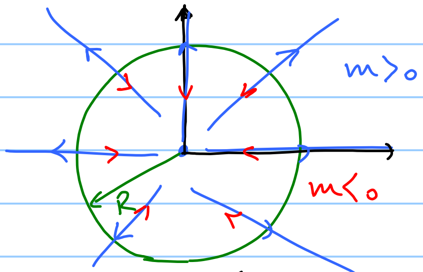


(جریان چاه یا چشمه) $\phi=m\ln r$ (2

$$\nabla^2\phi=\underset{-\frac{m}{r^2}}{\cancel{\frac{\partial^2\phi}{\partial r^2}}}+\frac{1}{r}\underset{\frac{m}{r}}{\cancel{\frac{\partial\phi}{\partial r}}}+\cancel{\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}}^{\,0}$$

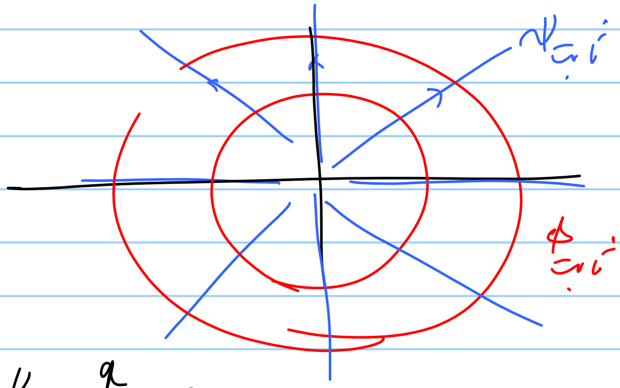
$$\Rightarrow -\frac{m}{r^2}+\frac{m}{r^2}=0 \Rightarrow 0=0 \quad \checkmark$$

$$v_r=\frac{\partial\phi}{\partial r}=\frac{m}{r} \quad , \quad v_\theta=\frac{\partial\phi}{\partial\theta}=0$$



$$q=\int v_r\,dl=\int_0^{2\pi}\underset{\frac{m}{R}}{\cancel{v_r}}\,R\,d\theta=\int_0^{2\pi}m\,d\theta=2\pi m$$

$$\Rightarrow m = \frac{q}{2\pi} \Rightarrow \phi = \frac{q}{2\pi}\ln r$$

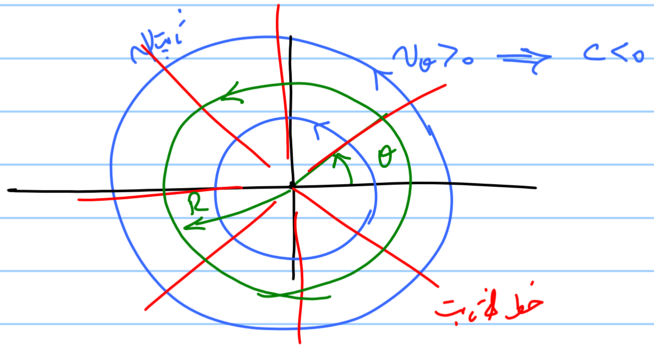


جریان $\psi = \frac{q}{2\pi}\theta$

$\phi = -C\theta$ (3

$$\nabla^2\phi = 0 \Rightarrow \underset{0}{\cancel{\frac{\partial^2\phi}{\partial r^2}}} + \frac{1}{r}\underset{0}{\cancel{\frac{\partial\phi}{\partial r}}} + \frac{1}{r^2}\underset{0}{\cancel{\frac{\partial^2\phi}{\partial\theta^2}}} = 0 \quad ✓$$

$$v_r = \frac{\partial\phi}{\partial r} = 0 \quad , \quad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{c}{r}$$



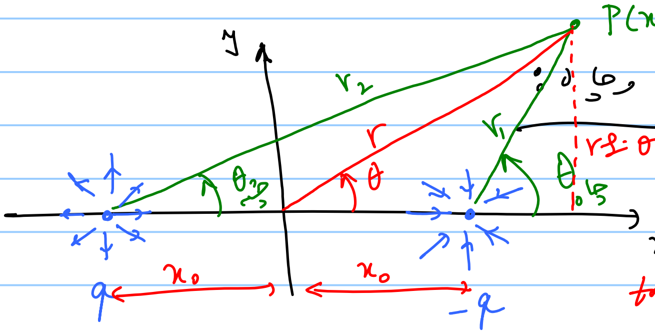
گردش Γ :

$$\Gamma = \oint \vec{v}\cdot d\vec{r} = \int_0^{2\pi}\left(-\frac{c}{\cancel{r}}\right)\cancel{r}\,d\theta = -2\pi C$$

$$\Rightarrow -C = \frac{\Gamma}{2\pi}$$

$$\Rightarrow \phi = \frac{\Gamma}{2\pi}\theta \Rightarrow \psi = -\frac{\Gamma}{2\pi}\ln r$$

ترکیب چشمه و چاه:



$$\tan\theta_1 = \frac{r\sin\theta}{r\cos\theta - x_0}$$

$$\tan\theta_2 = \frac{r\sin\theta}{r\cos\theta + x_0}$$

$$\psi_p = \psi_{\text{چاه}} + \psi_{\text{چشمه}} = -m\theta_{\text{چاه}} + m\theta_{\text{چشمه}}$$

$$= -m(\theta_1 - \theta_2)$$

$$\Rightarrow -\frac{\psi_p}{m} = \theta_1 - \theta_2$$

$$\Rightarrow \tan\left(-\frac{\psi}{m}\right) = \tan(\theta_1 - \theta_2)$$

$$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}$$

$$\Rightarrow \tan\left(-\frac{\psi}{m}\right) = \frac{r\sin\theta}{r\cos\theta - x_0} - \frac{r\sin\theta}{r\cos\theta + x_0}$$

$$= \frac{2m x_0 r \sin\theta}{r^2 - x_0^2}$$

$$\xRightarrow{\text{ترکیب چشمه و چاه}} \boxed{\psi = -m\tan^{-1}\left(\frac{2x_0 r\sin\theta}{r^2 - x_0^2}\right)} \quad (*)$$

$$\phi = K\frac{\cos\theta}{r} \qquad (4$$

$$\nabla^2\phi = 0 \Rightarrow ✓ \qquad \boxed{\Rightarrow \psi = -K\frac{\sin\theta}{r}}$$

اگر در ترکیب چاه و چشمه در شکل بالا، فاصله بین چاه و چشمه به سمت صفر میل داده شود

همزمان $m \to \infty$ میل داده شود:

$$(*) \Rightarrow \psi = -\frac{2\overset{q/2\pi}{\cancel{m}} x_0 \sin\theta}{r} = -\frac{2q x_0 \sin\theta}{2\pi r}$$

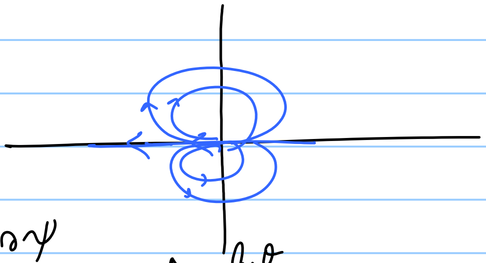
$$\begin{matrix} m \to \infty \\ x_0 \to 0 \end{matrix} \qquad\qquad \begin{matrix} q \to \infty \\ x_0 \to 0 \end{matrix}$$

اگر $2qx_0 = \Lambda$

$$\begin{matrix} x \to 0 \\ q \to +\infty \end{matrix} \qquad \boxed{\Rightarrow \psi = -\overset{K}{\frac{\Lambda}{2\pi}}\frac{\sin\theta}{r}}$$

$$\Rightarrow \boxed{\phi = \frac{\Lambda}{2\pi}\frac{\cos\theta}{r}}$$

جریان دابلت با
″ مزدوج

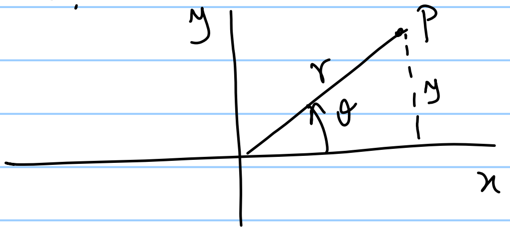


$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -\frac{\Lambda}{2\pi}\frac{\cos\theta}{r^2}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\frac{\Lambda}{2\pi}\frac{\sin\theta}{r^2}$$

برای تعیین خطوط جریان دابلت

$$\psi_{\text{دابلت}} = \text{Const.} \Rightarrow -\frac{\Lambda}{2\pi}\frac{\sin\theta}{r} = \psi = \text{Const.}$$



$$\sin\theta = \frac{y}{r}$$

$$\Rightarrow \psi = -\frac{\Lambda}{2\pi}\frac{y}{r^2} = -\frac{\Lambda}{2\pi}\frac{y}{x^2+y^2}$$

$$\Rightarrow x^2+y^2+\frac{\Lambda y}{2\pi\psi} = 0$$

$$\Rightarrow x^2+\left(y+\frac{\Lambda}{4\pi\psi}\right)^2 = \left(\frac{\Lambda}{4\pi\psi}\right)^2$$

معادله دایره