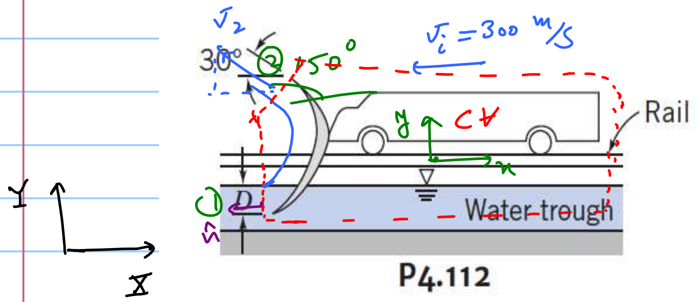
سوالات ۱ - سه‌شنبه ۹۹/۲/۲۳



P4.112

$L = 800$ m

$w = 0.3$ m

$M = 8000$ kg

$F_{D_i} = 90$ kN

$F_D = KV^2$

$V_f = 100$ m/s

$D = ?$

$$\underset{F_D}{\cancel{F_{S_x}}} + \underset{0}{\cancel{F_{B_x}}} - \int_{CV} \underset{-\frac{dV}{dt}}{\cancel{a_{rel_x}}}\,\rho\, d\forall = \overset{\simeq 0}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \int_{CS} u\rho\vec{V}\cdot\hat{n}\, dA$$

$$\Rightarrow KV^2 - M\left(-\frac{dV}{dt}\right) = V_1(-\rho V_1)\underset{DW}{\cancel{A_1}} + (-V_2\cos\theta)(\rho V_2)\underset{?}{\cancel{A_2}}$$

پیوستگی : $V_1A_1 = V_2A_2$

برنولی : $\cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + \cancel{gz_1} = \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + \cancel{gz_2} \Rightarrow V_1 = V_2 = V$

$\Rightarrow A_1 = A_2 = Dw = A$

$$\Rightarrow M\frac{dV}{dt} = -\rho V^2 A - \rho V^2\cos\underset{30^\circ}{\cancel{\theta}}\,A - KV^2$$

K تعیین : $F_{D_i} = KV_i^2 = 90$ kN

$\Rightarrow K(300)^2 = 90\times10^3 \Rightarrow K = 1$ ✓

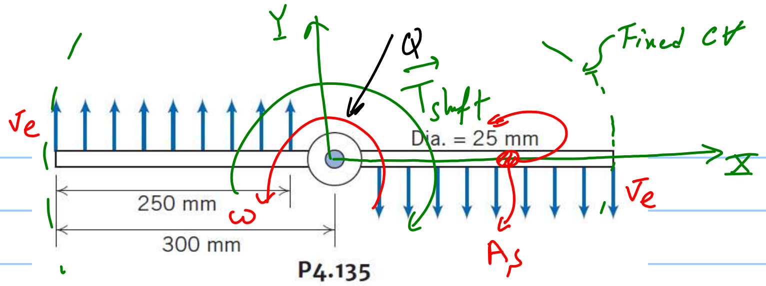
$$\Rightarrow \frac{dV}{dt} = -\boxed{\frac{[\rho A(1-\cos\theta)-1]}{M}}^{\,b} V^2$$

$$\Rightarrow \frac{dV}{V^2} = -b\,dt \Rightarrow \int_{V_i=300}^{V_f=100}\frac{dV}{V^2} = -\int_0^{t_f} b\,dt$$

$V(t)$ ✓

$\Rightarrow \boxed{t_f = ✓}$

$\underset{800}{\cancel{L}} = \int_0^{t_f} V\,dt \Rightarrow A$ ✓ $\Rightarrow D$ ✓



P4.135

$b = 2.5$ mm $R_i = 50$ mm , $R_o = 300$ mm , $Q = 3$ L/s

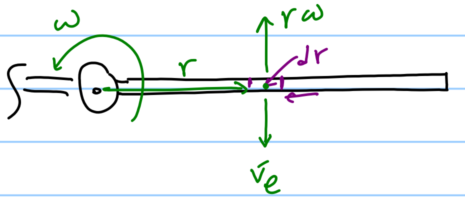
if $\omega = 0$, $T_{shaft} = ?$

if $T_{shaft} = 0$, $\omega = ?$

$$\cancelto{0}{\vec r \times \vec F_s} + \cancelto{0}{\int_{CV} \vec r \times \vec g \rho \, d\forall} + \vec T_{shaft} = \cancelto{0}{\frac{\partial}{\partial t} \int_{CV} \vec r \times \vec v \rho \, d\forall}$$

$$+ \int_{CS} \vec r \times \vec v \rho \vec v \cdot \hat n \, dA$$

$$\Rightarrow \vec T_{shaft} = \int_{CS} \vec r \times \vec v \rho \vec v \cdot \hat n \, dA$$, $\vec v =$ نسبت به دستگاه اینرسی



$$\Rightarrow \vec V = (r\omega - V_e)\hat j$$

$$\Rightarrow \vec T_{shaft} = \int_{CS} [\underbrace{r \hat i \times (r\omega - V_e)\hat j}_{\hat k}] \rho \vec v \cdot \hat n \, dA$$

$$\rho \vec v \cdot \hat n \, dA = \rho V_e b \, dr$$

$$= \left[2 \int_{R_i}^{R_o} (r^2 \omega - r V_e) \rho V_e b \, dr \right] \hat k$$

$$= \left[2 \rho V_e b \underline{\underline{\omega}} \frac{R_o^3 - R_i^3}{3} - 2 \rho V_e^2 b \frac{R_o^2 - R_i^2}{2} \right] \hat k$$

پیوستگی $Q = V_e [2b(R_o - R_i)] \Rightarrow \boxed{2 V_e b = \frac{Q}{R_o - R_i}}$

$$\Rightarrow \vec T_{shaft} = \rho Q \left[\frac{1}{3} \frac{R_o^3 - R_i^3}{R_o - R_i} \omega - V_e \frac{\cancel{R_o^2 - R_i^2}}{2 \cancel{(R_o - R_i)}} \right] \hat k$$

$$= \rho Q \left[\frac{1}{3} \frac{R_o^3 - R_i^3}{R_o - R_i} \omega - V_e \cancelto{R}{\frac{R_o + R_i}{2}} \right] \hat k$$

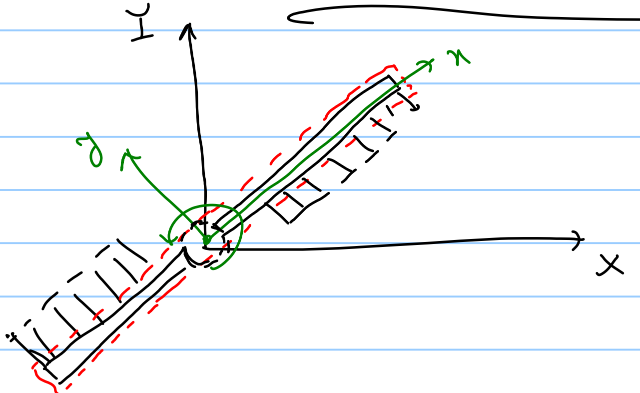
$$= \rho Q \left[\frac{1}{3} \frac{R_o^3 - R_i^3}{R_o - R_i} \omega - V_e R \right] \hat k$$ ✓

$$\text{if } \omega=0 \Rightarrow \vec{T}_{shaft} = -\rho Q V_e \bar{R}\,\hat{k}$$ ✓

$$\text{if } \vec{T}_{shaft}=0 \Rightarrow \frac{1}{3}\frac{R_o^3-R_i^3}{R_o-R_i}\,\omega = V_e \bar{R}$$

$$\Rightarrow \omega = \frac{3V_e\bar{R}(R_o-R_i)}{R_o^3-R_i^3}$$

تکمیل مسئله با استفاده از حجم کنترل دوار :



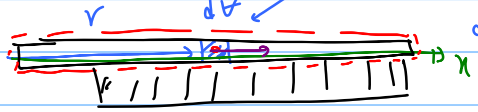
$$\cancel{\vec{r}\times\vec{F}_s}^{\,0} + \int_{C\forall}\cancel{\vec{r}\times\vec{g}}^{\,0}\,\rho\,d\forall + \vec{T}_{shaft} - \int_{C\forall}\vec{r}\times[2\vec{\omega}\times\vec{v}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+\cancel{\dot{\vec{\omega}}\times\vec{r}}^{\,0}]\rho\,d\forall$$

$$=\cancel{\frac{\partial}{\partial t}\int_{C\forall}\vec{r}\times\vec{v}\,\rho\,d\forall}^{\,0} + \int_{Cs}\vec{r}\times\vec{v}\,\rho\,\vec{v}\cdot\hat{n}\,dA$$

$$\Rightarrow \vec{T}_{shaft} - \int_{C\forall}\vec{r}\times[2\vec{\omega}\times\vec{v}+\underline{\vec{\omega}\times(\vec{\omega}\times\vec{r})}]\rho\,d\forall$$

$$=\int_{Cs}\vec{r}\times\vec{v}\,\rho\,\vec{v}\cdot\hat{n}\,dA$$

$$\int_{\boxed{C\forall}}\vec{r}\times[2\vec{\omega}\times\vec{v}]\rho\,d\forall = \int_{①\ [illegible]\ \forall}\vec{r}\times[2\vec{\omega}\times v]\rho\,d\forall + \int_{②\ [illegible]\ \forall} —$$



$$d\forall = A_s\,dr$$

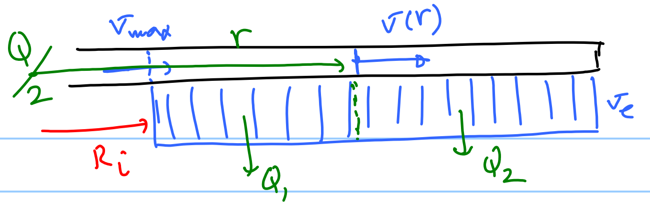
$$\int_0^{R_o}\underbrace{r\hat{i}\times[\underbrace{2\omega\hat{k}\times v\hat{i}}_{\hat{j}}]}_{\hat{k}}\rho A_s\,dr$$

$$=\int_0^{R_o}2\omega\rho A_s\,r\,v\,dr$$

$v=V_{max}=\text{const}\ 0<r<R_i$

$v=v(r)\quad R_i<r<R_o$

$=?$



$$\frac{Q}{2} = Q_1 + Q_2 = V_e\, b(r-R_i) + Q_2$$

$$\Rightarrow Q_2 = \frac{Q}{2} - V_e b(r-R_i)$$

(ازطرفی) $Q_2 = V(r)\, A_s \Rightarrow V(r) = \left[\frac{Q}{2A_s} - \frac{V_e b}{A_s}(r-R_i)\right]\hat{i}$

$r_i < r < r_o$ برای

همچنین : $V = V_{max} = \dfrac{Q}{2A_s}$ $\quad 0 < r < r_i$

$$\Rightarrow \int_{\forall_1} = \int_0^{R_i} r\hat{i} \times \left[\cancel{2}\omega\hat{k} \times \frac{Q}{\cancel{2}\cancel{A_s}}\hat{i}\right] \rho \cancel{A_s}\, dr$$

$$+ \int_{R_i}^{R_o} r\hat{i} \times \left[\underbrace{2\omega\hat{k} \times \left[\frac{Q}{2\cancel{A_s}}}_{} - \frac{V_e b}{\cancel{A_s}}(r-R_i)\right]\hat{i}\right] \rho \cancel{A_s}\, dr$$

$$= \left[\cancel{\rho\omega Q \frac{R_i^2}{2}} + \rho\omega Q\left(\frac{R_o^2 \cancel{- R_i^2}}{2}\right) - 2\rho\omega V_e b \frac{R_o^3 - R_i^3}{3} + 2\rho\omega V_e b R_i \frac{R_o^2 - R_i^2}{2}\right]\hat{k}$$

$V_e b = \dfrac{Q}{2(R_o - R_i)}$ با ترجه به اینکه

$$\Rightarrow \int_{\forall_1} = \rho\omega Q \frac{R_o^2}{2} - \rho\omega \frac{Q}{R_o - R_i} \frac{R_o^3 - R_i^3}{3} + \rho\omega \frac{Q}{R_o - R_i} R_i \frac{R_o^2 - R_i^2}{2}$$

$$\int_{\forall_2} = \quad 〃 \qquad 〃 \qquad 〃$$

$$\int_{CV} \vec{r} \times [2\vec{\omega} \times \vec{V}] \rho\, d\forall = \rho\omega Q\left[R_o^2 - \frac{2(R_o^3 - R_i^3)}{3(R_o - R_i)} + R_i(R_o + R_i)\right]\hat{k}$$

$$= \rho\omega Q\left(R_o^2 + R_i^2 + R_o R_i - \frac{2}{3}\frac{R_o^3 - R_i^3}{R_o - R_i}\right)$$

$$\int_{CV} \underbrace{\underset{\hat{i}}{\vec{r}} \times \underbrace{\left[\underset{\hat{k}}{\omega} \times \underbrace{(\underset{\hat{k}}{\vec{\omega}} \times \underset{i}{\vec{r}})}_{\hat{j}}\right]}_{-\hat{i}}}_{0} \rho\, d\forall = 0$$

$$\int_{CV} \vec{r}\times\vec{v}\,\rho\vec{v}\cdot\hat{n}\,dA = -2\int_{R_i}^{R_o} r V_e\,\rho V_e b\,dr\,\hat{k}$$

$$\left(V_e b = \frac{Q}{2(R_o-R_i)}\right) \qquad = -\rho V_e^2 b\,(R_o^2-R_i^2)\,\hat{k}$$

$$= -\rho Q V_e \underbrace{\frac{R_o+R_i}{2}}_{\bar{R}} = -\rho Q V_e \bar{R}\,\hat{k}$$

جایگذاری :

$$\Rightarrow \vec{T}_{shaft} - \rho\omega Q\left(R_o^2+R_i^2+R_oR_i-\frac{2}{3}\frac{R_o^3-R_i^3}{R_o-R_i}\right)\hat{k} = -\rho Q V_e\bar{R}\,\hat{k}$$

$$\Rightarrow \boxed{\vec{T}_{shaft} = \rho Q\left[\left(R_o^2+R_i^2+R_oR_i-\frac{2}{3}\frac{R_o^3-R_i^3}{R_o-R_i}\right)\omega - \underline{V_e\bar{R}}\right]\hat{k}}$$

① حجم کنترل دوار

با استفاده از حجم کنترل ثابت بدست آوردیم :

$$T_{shaft} = \rho Q\left[\frac{1}{3}\frac{R_o^3-R_i^3}{R_o-R_i}\omega - \underline{V_e\bar{R}}\right]\hat{k}$$

حجم کنترل ثابت

میتوان نشان داد که :

$$R_o^2+R_i^2+R_oR_i = \frac{R_o^3-R_i^3}{R_o-R_i}$$

$$\Rightarrow \underbrace{T_{shaft} = \rho Q\left[\frac{1}{3}\frac{R_o^3-R_i^3}{R_o-R_i}\omega - V_e\bar{R}\right]\hat{k}} \checkmark$$

دوار