Note Title 5/10/2020

سیالات ۱ - کنفرانس ۹۹/۲/۲۱

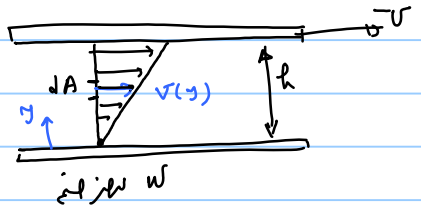
دبی جرمی $\dot{m} = \int_A \rho \vec{V} \cdot \hat{n} \, dA$

$\equiv$ Mass Flux

شار مومنتم $mf_x = \int_A u \rho \vec{V} \cdot \hat{n} \, dA$

شار انرژی جنبشی $Kef = \int_A \frac{V^2}{2} \rho \vec{V} \cdot \hat{n} \, dA$

4.12)

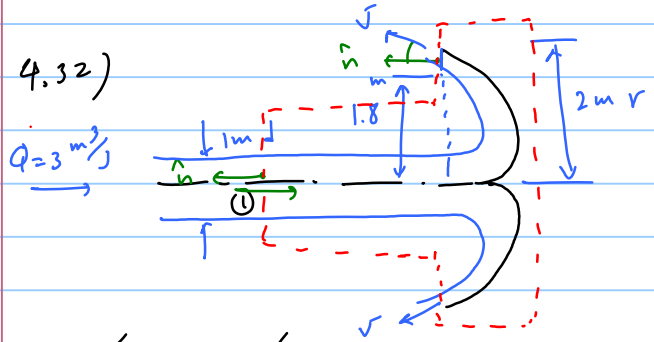


$$Kef = \int_0^h \frac{V^2}{2} \rho V w \, dy \quad , \quad V(y) = \frac{y}{h} V$$

$$= \int_0^h \frac{V^2/h^2 \; y^2}{2} \rho \frac{V}{h} y w \, dy$$

$$= \frac{1}{2} \rho w \frac{V^3}{h^3} \frac{h^4}{4} = \frac{1}{8} \rho w h V^3$$

4.32)



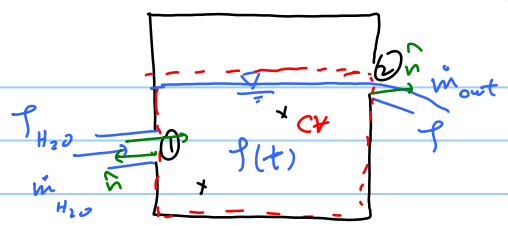
بقای جرم: $\frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = 0$

$\pi(2^2 - 1.8^2)$

$$\int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = 0 = -\rho V_1 A_1 + \rho V_2 \cos 45^\circ A_2 = 0$$

$Q = 3 \, m^3/s$

4.43)



$$\frac{\partial}{\partial t}\int_{CV}\rho\,dV+\int_{CS}\rho\vec{V}\cdot\hat{n}\,dA=0$$

$$\frac{\partial}{\partial t}\int_{CV}\rho\,dV=\frac{\partial}{\partial t}\rho\int_{CV}dV=V\frac{\partial\rho}{\partial t}$$

$$\int_{CS}\rho\vec{V}\cdot\hat{n}\,dA=-\rho_{H_2O}\underbrace{V_1A_1}_{Q_1}+\rho\underbrace{V_2A_2}_{Q_2}=Q(\rho-\rho_{H_2O})$$

$$\text{conservation of mass}\Rightarrow Q_1=Q_2=Q=\frac{\dot{m}_{H_2O}}{\rho_{H_2O}}=\frac{\dot{m}_f}{\rho(t)}$$

$$\text{conservation of mass}\Rightarrow V\frac{\partial\rho}{\partial t}+\frac{\dot{m}_{H_2O}}{\rho_{H_2O}}(\rho-\rho_{H_2O})=0$$

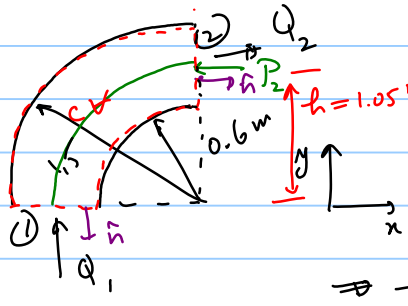
$$\Rightarrow\frac{\partial\rho}{\partial t}=-\frac{\dot{m}_{H_2O}}{\rho_{H_2O}V}(\rho-\rho_{H_2O})$$

$$\Rightarrow\int_{\rho_i}^{\rho_f}\frac{d\rho}{\rho-\rho_{H_2O}}=-\int_0^t\frac{\dot{m}_{H_2O}}{\rho_{H_2O}V}\,dt$$

$$\Rightarrow\ln\frac{\rho_f-\rho_{H_2O}}{\rho_i-\rho_{H_2O}}=-\frac{\dot{m}_{H_2O}}{\rho_{H_2O}V}t$$

$$\Rightarrow t=-\frac{\rho_{H_2O}V}{\dot{m}_{H_2O}}\ln\frac{\rho_f-\rho_{H_2O}}{\rho_i-\rho_{H_2O}}$$

4.56)



$P_1=345$ kPa

$$\text{conservation of mass}\Rightarrow\frac{\partial}{\partial t}\int_{CV}\rho\,dV+\int_{CS}\rho\vec{V}\cdot\hat{n}\,dA=0$$

$$\Rightarrow-\rho V_1A_1+\rho V_2A_2=0$$

$\Rightarrow -Q_1 + Q_2 = 0 \Rightarrow Q_1 = Q_2 = Q$

x-mom: $F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall + \int_{CS} u \rho \vec{V}\cdot\hat{n}\, dA$

$R_x - P_2 A_2 = u_2 (-\rho u_2) A_2$

$\Rightarrow R_x = (P_2 + \rho u_2^2) A_2$ $\quad u_2 = \frac{Q}{A}$

$P_2 = P_1 - \rho g h$ (34.6)

y-mom: $F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v \rho \, d\forall + \int_{CS} v \rho \vec{V}\cdot\hat{n}\, dA$

$\int_{CV} g_y \rho \, d\forall$ $\quad F_{B_y} = -\rho \forall g$ (وزن السائل)

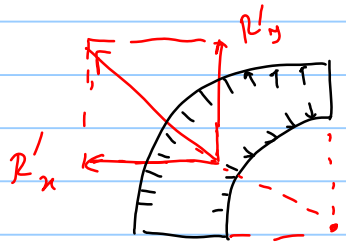
$R_y + P_1 A_1 = v_1 (-\rho v_1) A_1$, $v_1 = \frac{Q}{A}$

$\Rightarrow R_y$

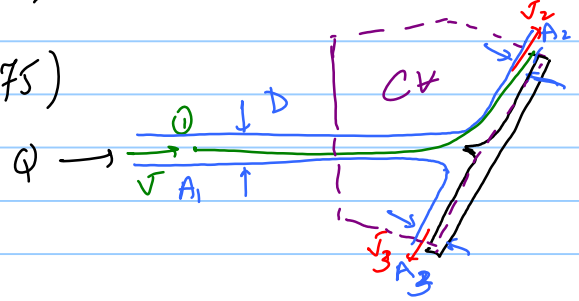
رد فعل القوة: $R'_x = -R_x$

$x \Rightarrow$

$R'_y = -R_y$



4.75)



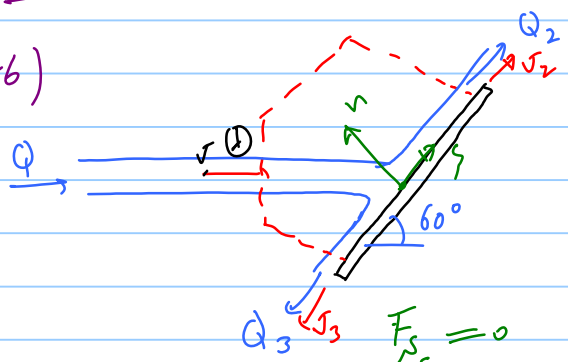
$\forall$: $V_3 A_3 = V_2 A_2 = Q/2$

$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2 \Rightarrow V_1 = V_2$

①, ③ $\Rightarrow V_1 = V_3 \Rightarrow V_1 = V_2 = V_3 = V$

$\Rightarrow R_x, R_y = \checkmark$

4.76)



$\frac{Q_2}{Q_1} = ?$

$\frac{Q_3}{Q_1} = ?$

x-mom: $F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u_x \rho \, dV + \int_{CS} u_x \rho \vec{V}\cdot\hat{n}\, dA$

$\Rightarrow \int_{CS} u_x \rho \vec{V}\cdot\hat{n}\, dA = 0 \Rightarrow (V\cos 60^\circ)(-\rho V_1)A_1 + V(\rho V_2)A_2 - V(\rho V_3)A_3 = 0$

$\Rightarrow -A_1 \cos 60^\circ + A_2 - A_3 = 0$ ✓

cont: $Q_1 + Q_2 + Q_3 = 0 \Rightarrow A_1 + A_2 + A_3 = 0$

$A_1 V_1 \quad A_2 V_2 \quad A_3 V_3$

A_2, A_3 ✓

$\frac{Q_2}{Q_1} = \frac{A_2}{A_1}$ ✓, $\frac{Q_3}{Q_1} = \frac{A_3}{A_1}$ ✓