## 120 Chapter 3 Gate-Level Minimization

- **3.12** Simplify the following Boolean functions:
  - (a)\*  $F(A, B, C, D) = \Pi(1, 3, 5, 7, 13, 15)$
  - (b)  $F(A, B, C, D) = \Pi(1, 3, 6, 9, 11, 12, 14)$
- **3.13** Simplify the following expressions to (1) sum-of-products and (2) products-of-sums:
  - (a)\* x'z' + y'z' + yz' + xy
  - (b) ACD' + C'D + AB' + ABCD
  - (c) (A' + B + D')(A' + B' + C')(A' + B' + C)(B' + C + D')
  - (d) BCD' + ABC' + ACD
- **3.14** Give three possible ways to express the following Boolean function with eight or fewer literals:

$$F = A'BC'D + AB'CD + A'B'C' + ACD'$$

- **3.15** Simplify the following Boolean function *F*, together with the don't-care conditions *d*, and then express the simplified function in sum-of-minterms form:
  - (a)  $F(x, y, z) = \Sigma(0, 1, 4, 5, 6)$
- (b)\*  $F(A, B, C, D) = \Sigma(0, 6, 8, 13, 14)$
- $d(x, y, z) = \Sigma(2, 3, 7)$

- $d(A, B, C, D) = \Sigma(2, 4, 10)$
- (c)  $F(A, B, C, D) = \Sigma(5, 6, 7, 12, 14, 15,)$  (d)  $F(A, B, C, D) = \Sigma(4, 12, 7, 2, 10,)$  $d(A, B, C, D) = \Sigma(3, 9, 11, 15)$   $d(A, B, C, D) = \Sigma(0, 6, 8)$
- **3.16** Simplify the following functions, and implement them with two-level NAND gate circuits:
  - (a) F(A, B, C, D) = AC'D' + A'C + ABC + AB'C + A'C'D'
  - (b) F(A, B, C, D) = A'B'C'D + CD + AC'D
  - (c) F(A, B, C) = (A' + C' + D')(A' + C')(C' + D')
  - (d) F(A, B, C, D) = A' + B + D' + B'C
- **3.17\*** Draw a NAND logic diagram that implements the complement of the following function:

$$F(A, B, C, D) = \Sigma(0, 1, 2, 3, 6, 10, 11, 14)$$

**3.18** Draw a logic diagram using only two-input NOR gates to implement the following function:

$$F(A, B, C, D) = (A \oplus B)'(C \oplus D)$$

- **3.19** Simplify the following functions, and implement them with two-level NOR gate circuits:
  - (a)\* F = wx' + y'z' + w'yz'
  - (b)  $F(w, x, y, z) = \Sigma(0, 3, 12, 15)$
  - (c) F(x, y, z) = [(x + y)(x = z)]'
- **3.20** Draw the multiple-level NOR circuit for the following expression:

$$CD(B+C)A + (BC'+DE')$$

**3.21** Draw the multiple-level NAND circuit for the following expression:

$$w(x + y + z) + xyz$$

- **3.22** Convert the logic diagram of the circuit shown in Fig. 4.4 into a multiple-level NAND circuit.
- **3.23** Implement the following Boolean function F, together with the don't-care conditions d, using no more than two NOR gates:

$$F(A, B, C, D) = \Sigma(2, 4, 10, 12, 14,)$$
  
$$d(A, B, C, D) = \Sigma(0, 1, 5, 8)$$

Assume that both the normal and complement inputs are available.

**3.24** Implement the following Boolean function *F*, using the two-level forms of logic (a) NAND-AND, (b) AND-NOR, (c) OR-NAND, and (d) NOR-OR:

$$F(A, B, C, D) = \Sigma(0, 4, 8, 9, 10, 11, 12, 14)$$

- 3.25 List the eight degenerate two-level forms and show that they reduce to a single operation. Explain how the degenerate two-level forms can be used to extend the number of inputs to a gate.
- **3.26** With the use of maps, find the simplest sum-of-products form of the function F = fg, where

$$f = abc' + c'd + a'cd' + b'cz'$$

and

$$g = (a + b + c' + d')(b' + c' + d)(a' + c + d')$$

- **3.27** Show that the dual of the exclusive-OR is also its complement.
- **3.28** Derive the circuits for a three-bit parity generator and four-bit parity checker using an odd parity bit.
- **3.29** Implement the following four Boolean expressions with three half adders:

$$D = A \oplus B \oplus C$$

$$E = A'BC + AB'C$$

$$F = ABC' + (A' + B')C$$

$$G = ABC$$

**3.30\*** Implement the following Boolean expression with exclusive-OR and AND gates:

$$F = AB'CD' + A'BCD' + AB'C'D + A'BC'D$$

- **3.31** Write a Verilog gate-level description of the circuit shown in
  - (a) Fig. 3.20(a)
- (b) Fig. 3.20(b)
- (c) Fig. 3.21(a)

- (d) Fig. 3.21(b)
- (e) Fig. 3.24
- (f) Fig. 3.25
- **3.32** Using continuous assignment statements, write a Verilog description of the circuit shown in
  - (a) Fig. 3.20(a)
- (b) Fig. 3.20(b)
- (c) Fig. 3.21(a)

- (d) Fig. 3.21(b)
- (e) Fig. 3.24
- (f) Fig. 3.25
- **3.33** The exclusive-OR circuit of Fig. 3.30(a) has gates with a delay of 3 ns for an inverter, a 6 ns delay for an AND gate, and a 8 ns delay for an OR gate. The input of the circuit goes from xy = 00 to xy = 01.
  - (a) Determine the signals at the output of each gate from t = 0 to t = 50 ns.
  - (b) Write a Verilog gate-level description of the circuit, including the delays.
  - (c) Write a stimulus module (i.e., a test bench similar to HDL Example 3.3), and simulate the circuit to verify the answer in part (a).
- **3.34** Using continuous assignments, write a Verilog description of the circuit specified by the following Boolean functions:

$$Out_1 = (A + B')C'(C + D)$$
  
 $Out_2 = (C'D + BCD + CD')(A' + B)$   
 $Out_3 = (AB + C)D + B'C$ 

Write a test bench and simulate the circuit's behavior.

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3.35\* Find the syntax errors in the following declarations (note that names for primitive gates are optional):

```
module Exmpl-3(A, B, C, D, F)
                                              // Line 1
inputs
                A, B, C, Output D, F,
                                              // Line 2
output
                                              // Line 3
                                              // Line 4
and
                g1(A, B, D);
not
                (D, A, C),
                                              // Line 5
OR
                (F, B; C);
                                              // Line 6
endmodule;
                                              // Line 7
```

**3.36** Draw the logic diagram of the digital circuit specified by the following Verilog description:

```
(a) module Circuit_A (A, B, C, D, F);
    input
                A, B, C, D;
     output
                F;
     wire
                w, x, y, z, a, d;
     or
                (x, B, C, d);
                (y, a ,C);
     and
     and
                (w, z, B);
                (z, y, A);
     and
    or
                (F, x, w);
    not
                (a, A);
    not
                (d, D);
   endmodule
(b) module Circuit_B (F1, F2, F3, A0, A1, B0, B1);
    output
                F1, F2, F3;
    input
                A0, A1, B0, B1;
                (F1, F2, F3);
    nor
                (F2, w1, w2, w3);
    or
                (F3, w4, w5);
    and
                (w1, w6, B1);
    and
                (w2, w6, w7, B0);
     or
    and
                (w3, w7, B0, B1);
                (w6, A1);
    not
    not
                (w7, A0);
                (w4, A1, B1);
    xor
                (w5, A0, B0);
    xnor
    endmodule
(c) module Circuit_C (y1, y2, y3, a, b);
    output y1, y2, y3;
    input a, b;
    assign y1 = a || b;
    and (y2, a, b);
     assign y3 = a \&\& b;
   endmodule
```

- **3.37** A majority logic function is a Boolean function that is equal to 1 if the majority of the variables are equal to 1, equal to 0 otherwise.
  - (a) Write a truth table for a four-bit majority function.
  - (b) Write a Verilog user-defined primitive for a four-bit majority function.
- **3.38** Simulate the behavior of *Circuit\_with\_UDP\_02467*, using the stimulus waveforms shown in Fig. P3.38.

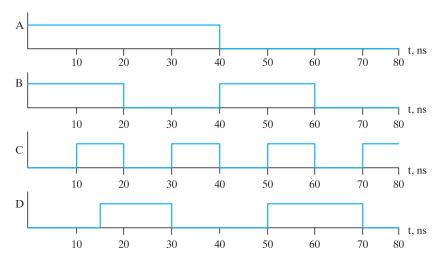


FIGURE P3.38
Stimulus waveforms for Problem 3.38

**3.39** Using primitive gates, write a Verilog model of a circuit that will produce two outputs, s and c, equal to the sum and carry produced by adding two binary input bits a and b (e.g., s = 1 and c = 0 if a = 0 and b = 1). (*Hint:* Begin by developing a truth table for s and c.)

## **REFERENCES**

- 1. Bhasker, J. 1997. A Verilog HDL Primer. Allentown, PA: Star Galaxy Press.
- **2.** CILETTI, M. D. 1999. *Modeling, Synthesis and Rapid Prototyping with the Verilog HDL*. Upper Saddle River, NJ: Prentice Hall.
- 3. HILL, F. J., and G. R. Peterson. 1981. *Introduction to Switching Theory and Logical Design*, 3rd ed. New York: John Wiley.
- **4.** *IEEE Standard Hardware Description Language Based on the Verilog Hardware Description Language* (IEEE Std. 1364-1995). 1995. New York: The Institute of Electrical and Electronics Engineers.
- **5.** Karnaugh, M. A Map Method for Synthesis of Combinational Logic Circuits. *Transactions of AIEE, Communication and Electronics*. 72, part I (Nov. 1953): 593–99.
- **6.** Kohavi, Z. 1978. *Switching and Automata Theory*, 2nd ed. New York: McGraw-Hill.